Orange County Math Circle
All-Girls Math Tournament
7th-8th Sprint Round

1. Clarisse flips a coin twice. What is the probability that the two flips yield different results?

2. A regular pentagon has one side with the value 6. What is the total perimeter?

3. Suppose that $a \star b = \frac{ab}{a+b}$. Compute $2 \star (3 \star 6)$.

4. The perimeter of a square with side length 6 is twice the perimeter of an equilateral triangle. What is the side length of the triangle?

5. Maya thinks of a number, multiplies it by 7, adds 14, and says the result out loud. If Maya says "84", what number was she originally thinking of?

6. Find the positive value of x if
$$\frac{1}{x} - 2 = x$$
.

7. A right triangle has side lengths x cm, 40 cm, 41 cm, and x is an integer. What is the area of the triangle?

8. Hannah pastes together four identical rectangles of length 4 and width 7 to form one large rectangle. What is the area of this rectangle?

9. A cylinder exists such that its radius, height, and volume are numerically equal. What is the radius of that cylinder?

10. Two chipmunks can build a treehouse in five hours. How much time will it take for ten chipmunks to build twenty treehouses?

11. A school has some books. All the books can be split evenly among 6 classrooms. They can also be split evenly among 10 classrooms. What is the smallest number of books that the school can have?

12. A positive integer n has a remainder of 3 when divided by 7 and a remainder of 7 when divided by 11. What is the remainder when n is divided by 77?

13. Let $f(x)$ be a quadratic function such that $f(x) = 3x^2 + 9x - 30$. Find $f(-5) + f(5)$.

14. Let x be a positive integer such that x has 7 factors, including x and 1. How many choices are there for x, if x cannot be greater than 50?

15. Two similar right triangles have the perimeters of 36 cm and 120 cm. If the shortest side of the smaller triangle is 9 cm and the difference between the other two sides is 3, what is the positive difference between the two hypotenuses?

16. Beth is waiting for her microwave to finish popping her popcorn. Popcorn in a microwave pops in a peculiar way: the first 10 kernels take 10 seconds to pop, and every 10 kernels after that take 10 more seconds to pop (so the second 10 kernels take 20 seconds, the third take 30 seconds, and so on). How much longer, in seconds, would Beth have to wait for a bag that contains 200 kernels to finish popping than a bag that contains 100 kernels?

17. Chad is running to school from his home, which is 2 miles away. His younger sister Katie started running 10 minutes beforehand, which also happened to be 30 minutes before school starts. The two siblings arrive just as school begins. How much faster was Chad running than Katie? Express your answer in miles per hour.

18. Express $\log_6 3$ in terms of x, where $x = \log_3 2$.

19. A *triangular number* is a number that can be written as $n(n+1)/2$ for a positive integer n. A *perfect square* is a number that can be written as m^2 for a positive integer m. Find the largest number less than 100 that is both a *triangular number* and a *perfect square*.

20. If 1 can of paint is enough covers 3 square meters, how many cans of paint are required to paint a right square pyramid with a height of 4 meters and a base with side length 6? You do not need to paint the bottom.

21. James tosses three six-sided dice and sums the three numbers that are on top. What is the probability that his sum is divisible by 12?

22. The first two terms of an arithmetic series are 5 and 10. The first two terms of a geometric series are also 5 and 10. What is the positive difference between the 5th terms of each series?

23. Find the area of a triangle enclosed by $y + 2 = 0$, $y = 3x + 10$, and $x = y + 8$.

24. For positive integers a and b, $\frac{1}{a} + \frac{2}{b} = \frac{1}{3}$. Find the largest possible value of ab.

25. Jessica is trying to fit square pegs into round holes. If the holes are perfectly circular, and the pegs just barely fit, what is the ratio of the diameter of the holes to the side length of the pegs?

26. A ball is shaped exactly like a sphere of radius r. When the ball has its outer layers ripped off, the core, a sphere with radius m, is revealed. Given that $\frac{m}{r} = \frac{3}{4}$, what is the ratio of the surface area of the core to the original surface area of the ball?

27. Grant wants to sell 28 cashews and an unknown amount of peanuts to you. A package of 100 cashews costs 3 dollars, and a package of 100 peanuts costs 4 dollars. Grant wants to make a profit of at least 20 cents for 100 nuts, so he sells his mixture of nuts for $3.92 per 100 nuts. What is the most amount of peanuts that Grant might have, based off of this information?

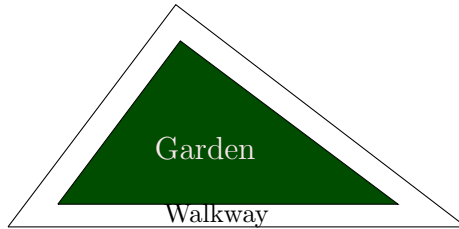28. Right triangle ABC has $AC = 8$ and $CB = 6$. M is the midpoint of AB. Pick point N on line CM with M between C and N such that $\angle CAB = \angle BAN$. Compute MN. Express your answer as a common fraction.
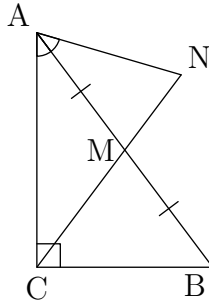
29. A walkway of width 1 surrounds a triangular garden, as shown. The garden's area is 54 and its perimeter is 36. Find the total area of walkway.

*Figure not drawn to scale

30. Two circles, O_1 and O_2, of radius 6 and 8, respectively, intersect at points P and Q, as shown. AB is a line segment that passes through P and with one end on each circle. AQ is tangent to $\odot O_2$, and BQ is tangent to $\odot O_1$. Find the area of $\triangle AQB$.