Orange County Math Circle
All-Girls Math Tournament
3rd-4th Sprint Round

1. You have 20 apples. You eat half of your apples. Your friend gives you 6 more apples. How many apples do you have?

2. What is the area of a triangle with height 7 and base 8?

3. The perimeter of a square with side length 6 is twice the perimeter of an equilateral triangle. What is the side length of the triangle?

4. Maya thinks of a number, multiplies it by 7, adds 14, and says the result out loud. If Maya says "84", what number was she originally thinking of?

5. Clarisse flips a coin twice. What is the probability that the two flips yield different results?

6. A regular pentagon has one side with the value 6. What is the total perimeter?

7. Suppose that $a \star b = \frac{ab}{a+b}$. Compute $2 \star (3 \star 6)$.

8. Find the number of divisors of 24, including 1 and 24.

9. Thirteen 2 by 2 squares are pasted together in a straight line. What is the perimeter of the resulting figure?

10. Allison has to solve 15 math problems for homework. 5 are easy problems, 5 are medium problems, and 5 are hard problems. If it takes her 2 minute to solve an easy problem, 4 minutes to solve a medium problem, and 7 minutes to solve a hard problem, how many minutes does it take her to finish her homework?

11. Kate is eating a square pizza. Her sister, Rachel, is eating a rectangular pizza with side lengths 4 and 9. Both sisters finish eating their pizzas and end up eating the same amount of pizza. What is the side length of the square pizza?

12. Hannah pastes together four identical rectangles of length 4 and width 7 to form one large rectangle. What is the area of this rectangle?

13. A school has some books. All the books can be split evenly among 6 classrooms. They can also be split evenly among 10 classrooms. What is the smallest number of books that the school can have?

14. Ara uses a keyboard with a total of 104 keys. She cannot read yet, so she randomly presses two keys, one after another. What is the probability that both keys are letters? (There are 26 letters in the alphabet).

15. A square and an equilateral triangle have side length 5. Michelle glues one edge of the equilateral triangle to one edge of the square, keeping the square and the triangle touching at only one edge. What is the perimeter of this new figure?

16. A triangle has two angles of 20° and 50°. What is the measure of the third angle?

17. A lake is filled with 20 fish. After every morning, 3 fish swim out of the lake and after every night, 2 fish swim in. After how many mornings will the lake first be empty?

18. Angle A in triangle ABC has an angle measure of 42 degrees. What is the average angle measure of the other two angles of ABC?

19. Two apples weigh as much as three oranges. Nine oranges weigh as much as ten pears. How many apples weigh as much as 40 pears?

20. $3x - 4y = 18$ and $4x - 3y = 10$. Compute $x - y$.

21. Two chipmunks can build a treehouse in five hours. How much time will it take for ten chipmunks to build twenty treehouses?

22. How many squares of side length 2 can fit inside a 5 by 7 rectangle with sides parallel to the sides of the rectangle and no overlap allowed?

23. A square frame for a square painting is made with a thickness of 1 inch. If the area of the frame is three times the area of the painting, what is the side length, in inches, of the painting?

24. Julia, Joey and, Jackie are going to Disneyland together. They each pay $158 for their ticket. When they get there, they spend $60 on lunch collectively and $72 on dinner collectively. If they split the bill equally for both meals and add their ticket price to their total, what did each person pay in total?

25. Isabella has a bag of 15 green, blue, and violet marbles. There are 9 blue marbles, and twice as many green marbles as violet marbles. If she randomly picks one marble, and without putting it back, randomly picks another marble, what is the probability that both marbles are green?

26. Jack has some 2 liter water bottles and some 3 liter water bottles. He wants to fill a pail of 17 liters while wasting no water. What is the least number of 2 liter bottles that Jack has to use?

27. I chose a two digit number. I then reversed the digits of the number: for example, from 19 to 91. I finally added these two numbers together and got 143. What is the sum of the digits of my original number?

28. A rectangle with integer side lengths has an area of 42. How many different possible values are there for its longer side?

29. Jason picks a positive integer. He multiplies his number by itself. Then, he subtracts 1. He gets the answer 399. What number did Jason pick?

30. Mohab is made up of a circle for his head, a square for his body, and 4 congruent rectangles for his arms and legs with no overlapping regions. The circle has a radius that is equivalent to each side of the square, 4 units. The dimensions of each rectangle are 6 units by 1 unit. What is Mohab's overall area?